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Efficient Open-loop Control for a Class of Stochastic Multistable Systems

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1. INTRODUCTION

The performance of certain nonlinear stochastic systems is deemed acceptable if, during a specified time interval, the systems have sufficiently low probabilities of escape from a preferred region of phase space. These probabilities can be reduced by using an appropriate control system.

We propose a Melnikov-based approach to achieving an efficient open-loop control. The approach is applicable to the wide class of multistable systems that have dissipation- and excitation-free counterparts possessing homoclinic or heteroclinic orbits. That class includes, e.g., the rf Josephson junction and the Duffing equation, and higher- and infinitely-dimensional systems.

We review the theoretical basis of our approach, use numerical simulations to test its effectiveness for the paradigmatic case of the stochastically excited Duffing equation, and discuss our results.

2. MELNIKOV PROCESSES AND EXITS FROM A WELL

The Melnikov approach is a technique providing necessary conditions for the occurrence of chaos in a class of dynamical systems. Until recently it was considered to be applicable only to deterministic systems. Following an extension from the case of periodic to the case of quasiperiodic excitation [1], Melnikov theory was further extended to the case of additive or multiplicative excitation by Gaussian, white, shot or dichotomous processes [2], [3].

Necessary conditions for the occurrence of chaos indicate the range of system parameters for which exits from preferred regions of phase space cannot occur. The Melnikov approach can thus help to study the exit problem for the wide class of systems.

For definiteness we consider the equation

$$\ddot{z} = -V'(z) + \epsilon[\gamma G(t) - \beta\dot{z})] \tag{1}$$

where β,γ are constants, $\beta>0$, and $V(z)$ is a potential function. We assume that: (i) the unperturbed system ($\epsilon=0$) is integrable; (ii) $V(z)$ has the shape of a multiple well so that the unperturbed system has a center at the bottom of each well and a saddle point at the top of the barrier between two adjacent wells. The stable and unstable manifolds emanating from the saddle point of the unperturbed system then coincide. Finally, we assume $G(t)$ is a random process. As a typical example we consider the Duffing equation with potential $V(z) = z^4/4 - z^2/2$, homoclinic orbits with coordinates $z_s(t)=(2)^{1/2}\operatorname{sech}(t)$, $\dot{z}_s(t)=(2)^{1/2}\operatorname{sech}(t)\tanh(t)$, and a modulus of the Fourier transform of the function $h(t)\equiv \dot{z}_s(-t)$

$$S(\omega)=(2)^{1/2}\pi\omega\operatorname{sech}(\pi\omega/2). \tag{2}$$

We also note for later use that $c \equiv \int_{-\infty}^{\infty} \dot{z}_s^2(\tau)d\tau = 4/3$.

Associated with Eq. 1 is a Melnikov process with the expression

$$M(t) = -\beta c + \gamma\int_{-\infty}^{\infty} h(\tau)G(t-\tau)d\tau. \tag{3}$$

Any realization of the Melnikov process represents the distance between the stable and unstable manifolds of Eq. 1 ($\epsilon\neq 0$) corresponding to a realization of the random process $G(t)$.

For any given system, increasing τ_e by using an open-loop control approach can be achieved by adding to the excitation $\epsilon\gamma G(t)$ a control force $\epsilon\gamma_c G_c(t)$, where γ_c has the same sign as γ. A trivial choice of the open-loop control force would be $G_c(t)\equiv -G(t)$. However, even if such a trivial open-loop control could be achieved, it would clearly inefficient. We propose to use the Melnikov approach to obtain a more efficient open-loop control force.

The mean zero upcrossing time, τ_M, of the Melnikov process is a measure of the mean time of exit from a well, τ_e, and is determined by the spectral density of the Melnikov process [4]. From Eq. 3 it follows that the spectral density of the Melnikov process for the uncontrolled system is $2\pi\Psi_M(\omega)=S^2(\omega)[2\pi\Psi(\omega)]$ where $S(\omega)$ is the modulus of the Fourier transform of $h(t)$, and $2\pi\Psi(\omega)$ is the spectral density of the random process $G(t)$.

For illustration purposes we consider the Duffing equation, for which $S(\omega)$ is given by Eq. 2, and the process $G(t)$ with

$$2\pi\Psi(\omega)=\begin{cases} 0.03990\ln(\omega)+0.12829 & 0.04\le\omega\le 0.4 \\ 0.05755\ln(\omega)+0.14493 & 0.4\le\omega\le 1.2 \\ -0.38301[\ln(\omega)]^2+1.06192\ln(\omega)-0.02941 & 1.2\le\omega\le 15. \end{cases}$$

To a first approximation this spectrum is representative of low-frequency fluctuations of the horizontal wind speed.

A graphic representation of $S^2(\omega)$ and $2\pi\Psi(\omega)S^2(\omega)$ shows that owing to the shape of $S^2(\omega)$, which plays the role of an admittance function, only part of the frequency components of the excitation $G(t)$ contribute significantly to the spectral density of the uncontrolled system's Melnikov process. We therefore propose the following approach. Instead of $G_c(t)\equiv -G(t)$, it would be more efficient to apply a control force obtained from the function $-G(t)$ by filtering out from this function those frequency components that do not contribute significantly to the spectral density $\Psi_M(\omega)$. The advantage of the proposed approach over the trivial approach $G_c(t)\equiv -G(t)$ is that, in general, it would reduce significantly the power needed for the system's control, while resulting in almost the same reduction of the ordinates (and the mean zero upcrossing rate) of the controlled system's Melnikov process. Given the dependence of the system's mean exit time on the Melnikov process mean zero upcrossing rate, the proposed approach, in spite of its reduced power needs, can be expected to result in almost the same improvement in the controlled system's behavior as the more onerous trivial approach.

The procedure just described -- like its trivial counterpart -- is unfeasible owing to limitations of practical control system. These limitations entail non-zero time lags between sensing of a signal and actuator response, as well as unavoidable inefficiencies of practical filters. We present next results of numerical simulations aimed at illustrating the potential of the proposed approach modified to account for non-zero time lags. Work on the role of other practical control system limitations is in progress.

3. SIMULATIONS AND DISCUSSION OF RESULTS

We considered the Duffing equation with $\epsilon=0.1$ and $\beta=0.45$ and examined the case where G(t) has the spectrum given earlier. We estimated by numerical simulation the mean exit rate for the uncontrolled system. We then estimated the mean exit rates for the system to which we applied control forces obtained by passing the function $-\epsilon\gamma_c G(t-\tau_0)$ through an ideal filter that suppresses all the Fourier components for $0\le\omega<\omega_1$ and $\omega>\omega_2$, and leaves the other components unchanged. We assumed (1) $\tau_0=0.1$ and (2) $\tau_0=0.5$. By inspecting the spectrum of the Melnikov process we chose $\omega_1=0.4$, $\omega_2=2.4$. Calculations showed that the final results being sought were affected insignificantly even for ω_1 as small as zero and ω_2 as large as the largest energy-containing frequency of the excitation spectrum. The results of the simulations are shown in Fig. 1, in which $\sigma\equiv\epsilon\gamma$.

For any given time lag τ_0, the relative effectiveness of the control with a Melnikov-based ideal filter is defined by the ratio ν/μ, where μ is the variance of the control force obtained by passing the excitation through the filter, and ν is the variance of the unfiltered excitation. The variances are measures of average power. Simple calculations yield $\nu/\mu=37.0$ for our case.

4. CONCLUSIONS

An open-loop approach to the control of a wide class of nonlinear stochastic systems was proposed with a view to achieving an efficient reduction of the mean exit rate from a potential well. It was shown that the Melnikov relative scale factors contain information that can be used for the design of filters suitable for that purpose. The degree to which an efficient Melnikov-based control can be achieved in practice depends upon the system's Melnikov scale factors, the spectrum of the excitation, and the quality of the filter design. Numerical simulations suggest that our approach can be effective in practice. However, the intent of this paper is merely to draw the attention of control specialists to the novel approach we propose, in the belief that -- whether used singly or as a component in a more complex control strategy -- it can become a useful addition to the current body of nonlinear control theory and practice.

5. ACKNOWLEDGMENTS

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6. REFERENCES

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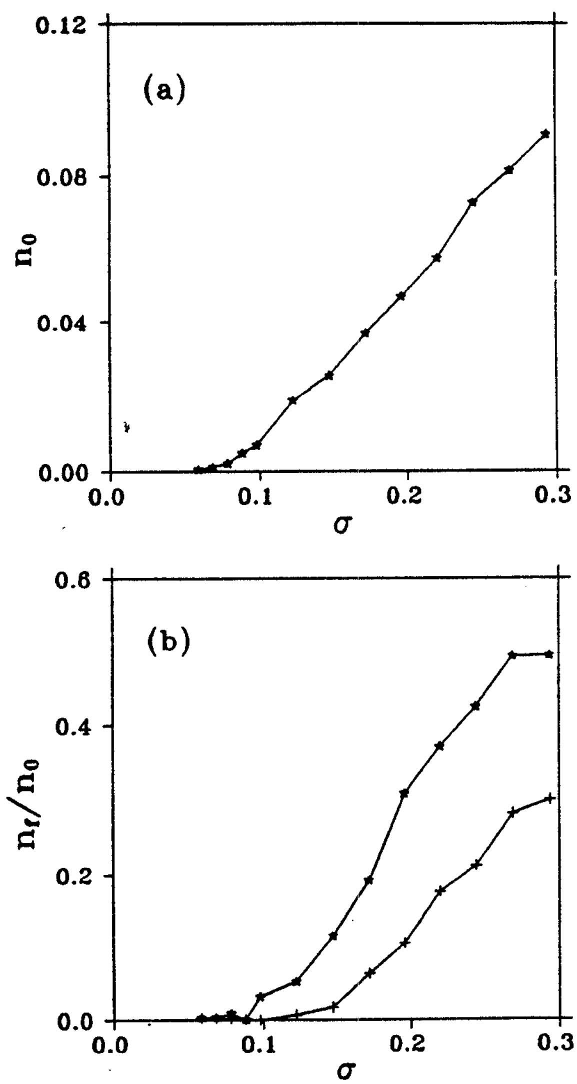

Fig. 1. (a) Mean exit rate n_0 for uncontrolled system with noise $\sigma\equiv\epsilon\gamma$; (b) ratio of controlled system's exit rate n_f to n_0 for time lag $\tau_0=0.1$ (lower curve) and $\tau_0=0.5$ (upper curve).